Family Support Materials

Exponent Review

This week your student will learn the rules for multiplying and dividing expressions with exponents. Exponents are a way of keeping track of how many times a number has been repeatedly multiplied. For example, instead of writing $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$, we can write 8^7 instead. The number repeatedly multiplied is called the base, which in this example is 8. The 7 here is called the exponent.

Using our understanding of repeated multiplication, we'll figure out several "rules" for exponents. For example, suppose we want to understand the expression $10^3 \cdot 10^4$. Rewriting this to show all the factors, we get $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$. Since this is really 7 10s multiplied together, we can write $10^3 \cdot 10^4 = 10^7$. By counting the repeated factors that are 10, we've added the exponents together (there are 3 of them, and then 4 more). This leads us to understanding a more general rule about exponents; when multiplying powers of the same base, we add the exponents together:

$$x^n \cdot x^m = x^{n+m}$$

Using similar reasoning, we can figure out that when working with powers of powers, we multiply the exponents together:

$$(x^n)^m = x^{n \cdot m}$$

These patterns will lead to other discoveries later on.

Here is a task to try with your student.

1. Jada and Noah were trying to understand the expression $10^4 \cdot 10^5$. Noah said, "since we are multiplying, we will get 10^{20}." Jada said, "But I don't think you can get 20 10s multiplied together from that." Do you agree with either of them?

2. Next, Jada and Noah were thinking about a similar expression, $(10^4)^5$. Noah said, "Ok this one will be 10^{20} because you will have 5 groups of 4." Jada said, "I agree it will be 10^{20}, but it's because there will be 4 groups of 5." Do you agree with either of them?

Solution:

1. Jada is correct. Rewriting $10^4 \cdot 10^5$ to show all the factors looks like $(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$. We can see that there are a total of 9 10s being multiplied. This helps us understand what's going on when we use the rule to write $10^4 \cdot 10^5 = 10^{4+5} = 10^9$.

2. This time, Noah is correct. When we look at $(10^4)^5$, the outside exponent of 5 tells us that there are 5 10^4s being multiplied together. So, $(10^4)^5 = 10^4 \cdot 10^4 \cdot 10^4 \cdot 10^4 \cdot 10^4$. This means there are 5 groups of 4 10s being multiplied together. We could write this out the long way as $(10^4)^5 = (10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)\,(10 \cdot 10 \cdot 10 \cdot 10)$ $(10 \cdot 10 \cdot 10 \cdot 10)$. This helps us understand what's going on when we use the rule to write $(10^4)^5 = 10^{4 \cdot 5} = 10^{20}$.

NAME ____________________ DATE __________ PERIOD ______

Scientific Notation

This week your student will use powers of 10 to work with very large or very small numbers. For example, the United States mint has made over 500,000,000,000 pennies. In order to understand this number, we have to count all of the zeros. Since there are 11 of them this means there are 500 billion pennies. Using powers of 10, we can write this as $5 \cdot 10^{11}$. The advantage to this way of writing the number is that we can see right away how many zeros there are (11), and more efficiently compare numbers when they are both written in this form. The same is true for small quantities. For example, a single atom of carbon weighs about 0.0000000000000000000000199 grams. If we write this using powers of 10, it becomes $(1.99) \cdot 10^{-23}$.

Not only do powers of 10 make it easier to write this number, but they also help avoid errors since it would be very easy to add or take away a zero when writing out the decimal without realizing! Writing numbers in this way is called scientific notation. We can use the exponent rules learned earlier to estimate and solve problems with scientific notation.

Here is a task to try with your student.

This table shows the top speeds of different vehicles.

Vehicle	Speed (kilometers per hour)
sports car	$(4.15) \cdot 10^2$
Apollo Command/Service Module	$(3.99) \cdot 10^4$
jet boat	$(5.1) \cdot 10^2$
autonomous drone	$(2.1) \cdot 10^4$

1. Order the vehicles from fastest to slowest.
2. The top speed of a rocket sled is 10,326 kilometers per hour. Is this faster or slower than the autonomous drone?
3. Estimate how many times as fast the Apollo Command/Service Module is than the sports car.

Solution:

1. The order is: Apollo CSM, autonomous drone, jet boat, sports car. Since all of these values are in scientific notation, we can look at the power of 10 to compare. The speeds of the Apollo CSM and autonomous drone both have the highest power of 10 (10^4), so they are fastest. The Apollo CSM is faster than the drone because 3.99 is greater than 2.1. Similarly, the jet boat is faster than the sports car because their speeds both have the same power of 10 (10^2) but 5.1 is greater than 4.15.

2. The autonomous drone is faster than the rocket sled. In scientific notation, the rocket sled's speed is $1.0326 \cdot 10^4$, and the drone's speed is $2.1 \cdot 10^4$ and 2.1 is greater than 1.0326.

3. To find how many times as fast the Apollo CSM is than the sports car, we are trying to find out what number times $4.15 \cdot 10^2$ equals $3.99 \cdot 10^4$. So we are trying to compute $\frac{3.99 \cdot 10^4}{4.15 \cdot 10^2}$. Since we are estimating, we can simplify the calculation to $\frac{4 \cdot 10^4}{4 \cdot 10^2}$. Using exponent rules and our understanding of fractions, we have $\frac{4 \cdot 10^4}{4 \cdot 10^2} = 1 \cdot 10^{4-2} = 10^2$, so the Apollo CSM is about 100 times as fast as the sports car!